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INFLUENCE OF TEMPERATURE CORRELATIONS ON PHASE KINETICS OF BOUNDARY FRICTION

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Using the Lorenz model for viscoelastic medium approximation the melting of ultrathin lubricant film is studied by friction between atomically flat surfaces. The fluctuations of lubricant temperature are taken into account defined by Ornstein-Uhlenbeck process. The phase portraits are defined corresponding to the different regions of dynamic phase diagram and determining system's kinetics. It is shown that the singular point, meeting the mode of dry friction, has indefinite character of stability. The other most probable steady-states of the system, corresponding to the stable and metastable sliding friction, are presented by the focus-type singular points in phase portraits. Consequently, the system can demonstrate damping oscillations near stable steady-states. The large extension of depicting trajectories near focuses along the axes of phase plane testifies to stability of sliding friction. Since maximums of distribution function, separated by the pronounced minimums of probability, correspond to the steady-state modes of friction, the transitions between them occur after large intervals of time.

Key words: viscoelastic medium, phase portrait, stick-slip friction.

Recently problems of sliding friction of flat solid surfaces attract a considerable attention at presence of thin film of lubricant between them [1,2]. It is conditioned by growth of need in low-friction elements in such high-tech components as computer storage systems, miniature engines, and aerospace devices. It is found experimentally that in the process of friction the liquid film becomes progressively thinner, at first its physical properties change gradually (quantitatively), and then the changes acquire the sharp (qualitative) character.

The boundary mode of friction is described in the proposed work realized in the case of ultrathin lubricant films with thickness less than four diameters of molecules at smooth surfaces or asperities, high loads, and low shear rates. It is characterized by the following changes of static (equilibrium) and dynamic properties of lubricant - simple unstructured Newtonian liquid [2]:

- non-fluidlike (non-Newtonian) properties: transition between liquid and solid phases, appearance of new liquid-crystalline states, epitaxially induced long-range ordering;

- tribological properties: absence of flow until yield point or critical shear stress reached, solidlike behaviour of liquid lubricant characterized by defect diffusion and dislocation motion, shear melting, boundary lubrication.

Experiments with mica, silica, metal oxide, and surfactant monolayer surfaces, between which organic liquids and aqueous solutions were placed, have shown that there are transformations between the different types of dynamic phases during sliding [2]. They manifest themselves in appearance of interrupted (stick-slip) friction, which is characterized by periodic transitions between two or more dynamic states during the stationary sliding and is the major reason for destruction and wear of frictional elements. Thus, molecularly thin lubricant films undergo more than one type of transition, that results to existence of different types of stick-slip mode of motion.

In works [3] a melting of ultrathin lubricant film by friction between atomically flat surfaces is represented as a result of self-organization of the shear stress and stain, and the temperature. The additive noises of these quantities are introduced for building the phase diagrams, where fluctuations intensities and frictional surfaces temperature define the domains of sliding, stick-slip, and dry friction [4]. In Ref. [5] the conditions are found at which the stick-slip friction regime corresponds to the intermittency mode inherent in self-organized criticality phenomenon. The dynamic phase diagram is studied taking into account correlated fluctuations of its temperature defined by Ornstein-Uhlenbeck process [6].

This paper, being the prolongation of Ref. [6], is devoted to investigation of kinetic modes of boundary friction using phase-plane method. The shear stress distribution function is studied and phase portraits are calculated for the cases of second-order and first-order transitions — the melting of amorphous and crystalline lubricant, respectively. The self-similar phase kinetics of lubricant film is investigated.

In the previous works [3] on the basis of rheological description of viscoelsatic medium the system of kinetic equations has been obtained, which define the mutually coordinated evolution of the elastic shear components of the stress σ and the strain ε, and the temperature T in ultrathin lubricant film during friction between atomically flat mica surfaces. Let us write these equations using the measure units

$$\sigma_s = \left(\frac{\rho c_v \eta_0 T_c}{\tau_T}\right)^{1/2}, \quad \varepsilon_s = \frac{\sigma_s}{G_0} \equiv \left(\frac{\tau_\varepsilon}{\tau_T}\right)^{1/2}\left(\frac{\rho c_v T_c \tau_\varepsilon}{\eta_0}\right)^{1/2}, \quad T_c \tag{1}$$

for variables σ, ε, T, respectively, where ρ is the mass density, c_v is the specific heat capacity, T_c is the critical temperature, $\eta_0 \equiv \eta(T = 2T_c)$ is the typical value of shear viscosity η, $\tau_T \equiv \rho l^2 c_v / k$ is the time of heat conductivity, l is the scale of heat conductivity, k is the heat conductivity constant, τ_ε is the relaxation time of matter strain, $G_0 \equiv \eta_0 / \tau_\varepsilon$:

$$\tau_\sigma \dot{\sigma} = -\sigma + g\varepsilon, \tag{2}$$

$$\tau_\varepsilon \dot{\varepsilon} = -\varepsilon + (T - 1)\sigma, \tag{3}$$

$$\tau_T \dot{T} = (T_e - T) - \sigma\varepsilon + \sigma^2 + \lambda(t). \tag{4}$$

Here the stress relaxation time τ_σ, the temperature T_e of atomically flat mica friction surfaces, and the constant $g = G / G_0$ are introduced, where G is the lubricant shear modulus. Equation (2) is reduced to the Maxwell-type equation for viscoelastic matter approximation by replacement ε/τ_σ on $\partial\varepsilon/\partial t$. As is known [1] the Maxwell-type equation is widely used in the theory of boundary friction. The relaxation behaviour of viscoelastic lubricant during the process of friction is described also by Kelvin-Voigt equation (3) [3, 7]. It takes into account the dependence of the shear viscosity on the dimensionless temperature $\eta = \eta_0 /(T - 1)$ [8]. Jointly, Eqs. (2) and (3) represent the new rheological model. It is worth noting that rheological properties of lubricant film are investigated experimentally for construction of phase diagram [2]. Equation (4) represents the heat conductivity expression, which describes the heat transfer from the friction surfaces to the layer of lubricant, the effect of the dissipative heating of a viscous liquid flowing under the action of the stress, and the reversible mechanic-and-caloric effect in linear approximation. Equations (2) - (4) coincide with the synergetic Lorenz system formally [9, 10], where the elastic shear stress acts as the order parameter, the conjugate field is reduced to the elastic shear strain, and the temperature is the control parameter. As is known this system can be used for description of the thermodynamic phase and the kinetic transitions.

The purpose of this work is to study the kinetics of boundary friction, described by the phase portraits, at introduction into Eq. (4) the stochastic source $\lambda(t)$ representing the Ornstein-Uhlenbeck process:

$$\langle\lambda(t)\rangle = 0,\ \langle\lambda(t)\lambda(t')\rangle = \frac{I}{\tau_\lambda}\exp\left(-\frac{|t-t'|}{\tau_\lambda}\right), \tag{5}$$

where I is the fluctuations intensity, τ_λ is the time of their correlation.

In Refs. [3] a melting of ultrathin lubricant film by friction between atomically flat mica surfaces has been represented as a result of action of spontaneously appearing elastic field of stress shear component caused by the heating of friction surfaces above the critical value $T_c = 1 + g^{-1}$. Thus, according to such approach the studied solidlike-liquidlike transition of lubricant film occurs due to both thermodynamic and shear melting. The initial reason for this self-organization process is the positive feedback of T and σ on ε [see Eq. (3)] conditioned by the temperature dependence of the shear viscosity leading to its divergence. On the other hand, the negative feedback of σ and ε on T in Eq. (4) plays an important role since it ensures the system stability.

According to this approach the lubricant represents a strongly viscous liquid that can behave itself similar to the solid — has a high effective viscosity and still exhibits a yield stress [2, 7]. Its solidlike state corresponds to the elastic shear stress $\sigma = 0$ because Eq. (2), describing the elastic properties at steady state $\dot{\sigma} = 0$, falls out of consideration. Equation (3), containing the viscous stress, reduces to the Debye law describing the rapid relaxation of the elastic shear strain during the microscopic time $\tau_\varepsilon \approx a/c \sim 10^{-12}$ s, where $a \sim 1$ nm is the lattice constant or the intermolecular distance and $c \sim 10^3$ m/s is the sound velocity. At that the heat conductivity equation (4) takes the form of simplest expression for temperature relaxation that does not contain the terms representing the dissipative heating and the mechanic-and-caloric effect of a viscous liquid.

At non-zero value of σ Eqs. (2)-(4) describes the above mentioned properties inherent in the liquidlike state of lubricant. Moreover, in accordance with Ref. [11] in the absence of shear deformations the temperature mean-square displacement is defined by equality $\left\langle u^2 \right\rangle = T / Ga$. The average shear displacement is found from the relationship $\left\langle u^2 \right\rangle = \sigma^2 a^2 / G^2$. The total mean-square displacement represents the sum of these expressions provided that the thermal fluctuations and the stress are independent. Above implies that the transition of lubricant from solidlike to fluidlike state is induced both by heating and under influence of stress generated by solid surfaces at friction. This agrees with examination of solid state instability within the framework of shear and dynamic disorder-driven melting representation in absence of thermal fluctuations. It is assumed that the film becomes more liquidlike and the friction force decreases with the temperature growth due to decreasing activation energy barrier to molecular hops. Besides, the friction force decreases with increasing velocity at the contact $V = l\partial\varepsilon / \partial t$ because the latter leads to the growth of the shear stress σ according to the Maxwell stress - strain ε relation: $\partial\sigma / \partial t = -\sigma / \tau_\sigma + G\partial\varepsilon / \partial t$.

This work is devoted to study of stochastic source $\lambda(t)$ influence on evolution of stress $\sigma(t)$. In accordance with experimental data for organic lubricant [2, 4] stress relaxation time at normal pressure is equal to $\tau_\sigma \sim 10^{-10}$ s. Since ultrathin lubricant film consists of less than four molecular layers, the temperature relaxes to the value T_e during the time satisfying to inequality $\tau_T << \tau_\sigma$. Therefore, we will suppose that conditions are fulfilled

$$\tau_\sigma \approx \tau_\varepsilon >> \tau_T, \tag{6}$$

at which lubricant temperature T follows to the change of shear components of stress σ and strain ε. Then, it is possible to select a small parameter and to put $\tau_T \dot{T} \cong 0$ in Eq. (4). As a result, we obtain the expression for temperature:

$$T = T_e - \sigma\varepsilon + \sigma^2 + \lambda(t). \tag{7}$$

Let us give to the system (2), (3), (7) more simple form, reducing it to the single equation for shear stress $\sigma(t)$. For this purpose it is necessary to express ε and T via σ. Differentiating with respect to time the equation for strain ε, that is obtained from (2), we get equation for $\dot{\varepsilon}$. Substituting these expressions for ε, $\dot{\varepsilon}$, and equality (7) in (3), we obtain evolution equation in canonical form of equation for nonlinear stochastic oscillator of the van der Pole generator type:

$$m\ddot{\sigma} + \gamma(\sigma)\dot{\sigma} = f(\sigma) + \phi(\sigma)\lambda(t), \tag{8}$$

where coefficient of friction γ, force f, amplitude of noise ϕ, and parameter m are defined by expressions

$$\gamma(\sigma) \equiv \frac{1}{g}\left[\tau_\varepsilon + \tau_\sigma\left(1 + \sigma^2\right)\right],\ f(\sigma) \equiv \sigma\left(T_e - 1 - g^{-1}\right) - \sigma^3\left(g^{-1} - 1\right),$$

$$\phi(\sigma) \equiv \sigma,\ m \equiv \frac{\tau_\sigma \tau_\varepsilon}{g}. \tag{9}$$

Let us find the distribution function of the stress σ. To that end we will use the method of effective potential [6], [12] — [14]. As a result, the Fokker-Planck equation is obtained:

$$\frac{\partial P}{\partial t}=-\frac{\partial}{\partial\sigma}\left(D^{(1)}P\right)+\frac{\partial}{\partial\sigma}\left(D^{(2)}\frac{\partial P}{\partial\sigma}\right). \tag{10}$$

It is expressed in terms of coefficients

$$D^{(1)}=\frac{1}{\gamma}\left\{f-I\phi^2\frac{\partial\gamma^{-1}}{\partial\sigma}-\phi\frac{\partial\phi}{\partial\sigma}\left(\frac{2I}{\gamma}+I\tau_\lambda\right)\right\}, \tag{11}$$

$$D^{(2)}=\frac{\phi^2}{\gamma}\left(\frac{I}{\gamma}+2I\tau_\lambda\right). \tag{12}$$

In the stationary case the solution of Eq. (10) leads to the distribution

$$P(\sigma)=Z^{-1}\exp\{-E(\sigma)\}, \tag{13}$$

which is fixed by effective potential

$$E(\sigma)=-\int_0^\sigma\frac{D^{(1)}(x)}{D^{(2)}(x)}dx \tag{14}$$

and normalization constant

$$Z=\int_0^\infty d\sigma\exp\left(\int_0^\sigma\frac{D^{(1)}(x)}{D^{(2)}(x)}dx\right) \tag{15}$$

The stationary shear stress is found from the extremum condition of distribution (13)

$$\frac{D^{(1)}(\sigma)}{D^{(2)}(\sigma)}=0. \tag{16}$$

According to Fig. 1 the distribution (13) has pronounced maximums whose position is determined by the set of parameters τ_σ, τ_ε, τ_λ, g, I, and T_e. At the small values of friction surfaces temperature T_e a single maximum is realized at point $\sigma=0$ meeting the dry friction mode. With T_e growth two maximums appear at points $\sigma=0$, and $\sigma\neq 0$, the first of them corresponds to the dry friction, the second one — to the sliding. The stick-slip friction mode is realized here characterized by transitions between the indicated stationary regimes. With further growth of T_e the zero maximum of $P(\sigma)$ disappears, and maximum at $\sigma\neq 0$ remains only, i.e. lubricant becomes liquidlike. The critical value of temperature T_e, providing transition of the system to the sliding friction, is defined by those quantities as the maximums of distribution function.

Apparently, the increase of the sheared surfaces temperature T_e transforms lubricant to the sliding friction mode. It can be understood considering Eq. (8) that describes the damping oscillations. Here, the surfaces temperature is included only in expression for driving force f, which increases with growth of T_e. As is known, the liquid can correspond to the oscillation mode with large amplitude, but solid can not. At the increasing value of effective force in (9) the amplitude of oscillations increases, and more long oscillation process is realized to the moment of establishment of the certain mode of friction in the system.

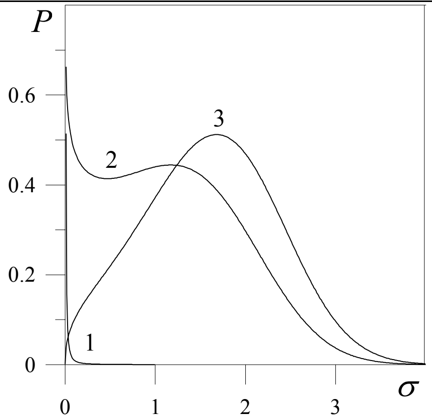

Fig. 1. The distribution function of shear stress for the second-order transition at $g = 0{,}2$, $\tau_\sigma = \tau_\varepsilon = 0{,}1$, $\tau_\lambda = 0{,}2$, and $I = 5$. The curves 1, 2, 3 correspond to the temperatures $T_e = 5, 16, 20$, respectively

For studying the dynamics of change of friction modes it is enough to represent distribution $P(\sigma)$ by position of its maximum $\tilde{\sigma}$. This is achieved by the use of path integrals formalism [15], within the framework of which the extreme values $\tilde{\sigma} = \tilde{\sigma}(t)$ of initial distribution function (13) evolve in accordance with the effective distribution

$$\Pi\{\dot{\tilde{\sigma}}, \tilde{\sigma}\} \propto \exp\left(-\int \Lambda\left(\dot{\tilde{\sigma}}, \tilde{\sigma}, t\right) dt\right) \tag{17}$$

where Onsager-Machlup function Λ, acting as the Lagrangian of Euclidean field theory, is the subject for determination.

Equation (10) can be transformed to the form:

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial \sigma}\left(\left(D^{(1)} + \frac{dD^{(2)}}{d\sigma}\right)P\right) + \frac{\partial^2}{\partial \sigma^2}\left(D^{(2)}P\right). \tag{18}$$

For finding of $\Lambda\left(\dot{\tilde{\sigma}}, \tilde{\sigma}, t\right)$ dependence we write down the differential Langevin equation

$$d\tilde{\sigma} = \left[D^{(1)} + \frac{dD^{(2)}}{d\sigma}\right]dt + \sqrt{2D^{(2)}}\, dW(t), \tag{19}$$

corresponding to Focker–Plank equation (18) [15]. Here, stochastic differential $dW(t)$ represents Winner process possessing properties of white noise:

$$\langle dW(t)\rangle = 0, \quad \left\langle (dW(t))^2 \right\rangle = dt\,. \tag{20}$$

The feature of stochastic equations is that differential $dW(t)$ can not be obtained by the simple division of Eq. (19) by $\sqrt{2D^{(2)}}$. To that end it is necessary to pass from a random process $\tilde{\sigma}(t)$ to white noise $x(t)$ related with the initial Jacobian $dx/d\tilde{\sigma} = \left(2D^{(2)}\right)^{-1/2}$. Then substitution of Eq. (19) in the Ito stochastic differential

$$dx = \frac{dx}{d\tilde\sigma} d\tilde\sigma + \frac{1}{2}\frac{d^2 x}{d\tilde\sigma^2}(d\tilde\sigma)^2, \tag{21}$$

taking into account (20), leads to expression

$$dx = \left[\frac{dx}{d\tilde\sigma}\left(D^{(1)} + \frac{dD^{(2)}}{d\tilde\sigma}\right) + \frac{d^2 x}{d\tilde\sigma^2} D^{(2)}\right] dt + \frac{dx}{d\tilde\sigma}\sqrt{2D^{(2)}}\, dW(t). \tag{22}$$

Here, the terms are neglected whose order exceeds $(dW(t))^2$. After reverse transition from white noise $x(t)$ to the initial process $\tilde\sigma(t)$ the equality is obtained

$$\frac{dW(t)}{dt} = \frac{\dot\sigma}{\sqrt{2D^{(2)}}} - \frac{2D^{(1)} + \left(D^{(2)}\right)'}{2\sqrt{2D^{(2)}}} \tag{23}$$

where the stroke stands for differentiation with respect to $\tilde\sigma$. Plugging this expression into Gaussian

$$\Pi \propto \exp\left\{-\frac{1}{2}\int\left(\frac{dW(t)}{dt}\right)^2 dt\right\},$$

and comparing with (17) we arrive at Lagrangian

$$\Lambda = \frac{1}{4}\frac{\dot{\tilde\sigma}^2}{D^{(2)}} - U \tag{24}$$

with potential energy

$$U = -\frac{\left(2D^{(1)} + \left(D^{(2)}\right)'\right)^2}{16 D^{(2)}} \tag{25}$$

It is substantial that such form of potential energy U does not coincide with effective potential (14). Consequently, for further considerations it is necessary to replace the expression U (25) on $E(\sigma)$ (14). In this case Eq. (24) describes the system behaviour in accordance with distribution (13) obtained above.

The system kinetics is defined by Euler–Lagrange equation

$$\frac{\partial\Lambda}{\partial\tilde\sigma} - \frac{\partial}{\partial t}\frac{\partial\Lambda}{\partial\dot{\tilde\sigma}} = \frac{\partial R}{\partial\dot{\tilde\sigma}} \tag{26}$$

Within the white noise presentation the dissipative function has the simplest form $R = \dot x^2/2$ and is transformed to

$$R = \frac{\dot{\tilde\sigma}^2}{4D^{(2)}} \tag{27}$$

with transition to the variable $\dot{\tilde\sigma} = \left(2D^{(2)}\right)^{1/2}\dot x$. Substituting in (26) the equalities (24), (14), (27), we arrive at differential equation

$$\ddot{\tilde\sigma} + \dot{\tilde\sigma}^2\frac{\left(D^{(2)}\right)'}{2D^{(2)}} + \dot{\tilde\sigma} - 2\frac{D^{(1)}}{D^{(2)}}D^{(2)} = 0. \tag{28}$$

Its study is described below based on the phase plane method $\left(\dot{\tilde\sigma}, \tilde\sigma\right)$.

Let us consider the steady-states at first. Supposing in (28) $\dot{\tilde{\sigma}} = 0$ the equation is obtained

$$\frac{D^{(1)}}{D^{(2)}} = 0 . \tag{29}$$

It coincides with the extremum condition of distribution (13). The distribution maximum corresponds to the minimum of effective potential, and the distribution minimum — to its maximum.

Consider kinetics of the system using the phase portraits defined by equation (28). The shear stress distribution dependences $P(\sigma)$ are shown in Fig. 1 for the different modes of friction for second-order phase transition. The curves 1, 2, 3 correspond the regions of dry (DF), stick-slip (SS), and sliding (SF) friction. The phase portraits are presented in Fig. 2 corresponding to the curves of Fig. 1.

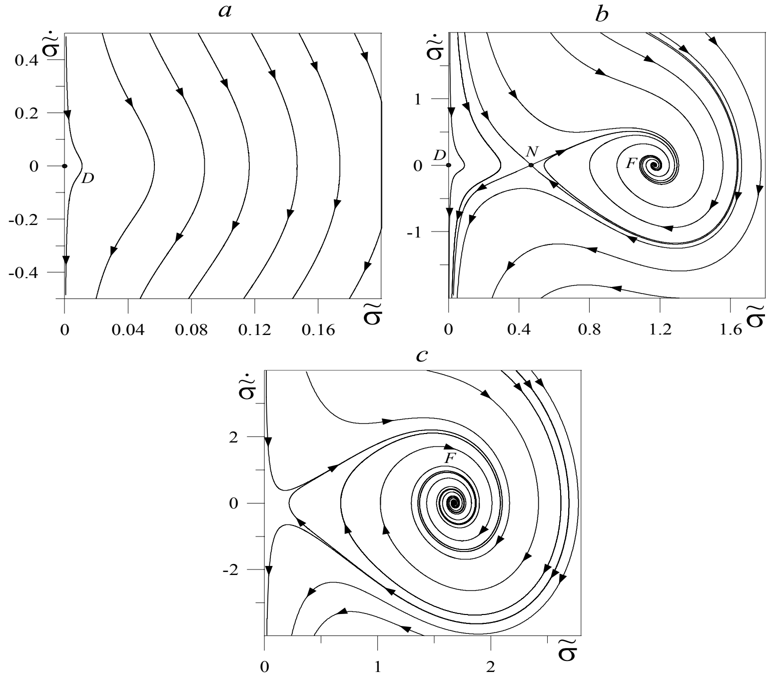

Fig. 2. The phase portraits corresponding to the parameters of Fig. 1: *a* – DF mode corresponds to the curve 1 in Fig. 1; *b* – SS – curve 2 in Fig. 1; *c* – SF – curve 3 in Fig. 1

The region of dry friction (DF) (Fig. 2, a) is characterized by a presence of one singular point *D*, which corresponds to the maximum of probability $P(\sigma)$ at $\sigma = 0$. This point is non-standard and requires interpretations. It is located at the origin of coordinates, and phase trajectories is curved around it, so that the system never comes to the value $\sigma = 0$, i.e. this point is not stationary. Consider the system behaviour at the arbitrary initial condition. According to the phase trajectories the system evolves to the zero value of stress. Thus, if in initial conditions the growth rate of stress is positive it, at first, decreases to the zero (during this time the stress increases). And then the stress value decreases asymptotically to the zero with the increase of its decrease rate. Presumably, the described situation meets the mode of explosive amorphization, at which the system transforms very rapidly in amorphous solidlike state. The circumstance that zero stress is never achieved testifies to divergence of probability $P(\sigma)$ at zero point. This is related with infinite growth of decrease rate of stress at going of the system to $\sigma = 0$. Let us assume that the system reaches the point at which decreasing rate becomes critical. In such case the increase of stress value is expected, which is accompanied by the change of rate sign and transition of the system in a positive phase plane region. Further, again the decreasing of stress occurs, and the described situation repeats oneself. Alternatively, the rate sign does not change, and only its decreasing takes place. This moves the system on a neighbouring phase trajectory, along which it approaches to the zero stress more quickly. As a result, in course of time the oscillation mode of dry friction is set in the vicinity of point $\sigma = 0$ at the arbitrary initial conditions. At that the oscillations amplitude is small, and lubricant has solidlike structure.

Phase portrait of the system, characterizing the region of stick-slip friction (SS), is shown in Fig. 2b. The three special points appear here: *D*, saddle *N*, and stable focus *F*. As well as above, point *D* is realized at the origin of coordinates and corresponds to the dry friction mode in the system. Saddle *N* meets the minimum of $P(\sigma)$, and is unstable stationary point. It is worth noting that at the initial value of shear stress on the right-hand side from point *N* and $\dot{\sigma} = 0$, the sliding mode of friction is set in the system during time. If the initial value of stress appears on left-hand side from saddle *N*, the dry friction is set in similar case. Thus, point *N* separates two maximums of distribution function $P(\sigma)$. Focus *F* corresponds to the non-zero maximum of stress distribution function, i.e., it describes the liquidlike state of lubricant. The corresponding to this point damping oscillations mean that lubricant becomes more liquidlike, and more solidlike periodically. However, the stable sliding friction is set always. Presumably, these oscillations are conditioned by the presence of noise.

The phase portrait corresponding to the sliding friction (SF) is shown in Fig. 2, c, which is characterized by one non-zero maximum of distribution function $P(\sigma)$. It is characterized by single singular point — the stable focus *F* corresponding to the stable sliding friction. This is confirmed also by large overexpansion of phase trajectories near it along axes of ordinates and abscissas. However, it is apparent that at the initial large value σ the system does not reach the point *F*, and approaches asymptotically to the zero stress value. This circumstance implies that conditions can be realized, at which the system will be near to the regime of dry friction. As described above, at reaching the value of critical rate, its sign changes and becomes positive. It is seen from the phase portrait, that in such case the system will pass to the mode of stable sliding friction.

Actually, the shear modulus, introduced (in terms of the relaxation time τ_σ) in Eq. (2), depends on the stress value. This leads to the transition of the elastic deformation mode to the plastic one. It takes place at characteristic value of the stress σ_p, which does not exceed the value σ_s (in other case the plastic mode is not manifested). For consideration of deformational defect of the modulus we will use $\tau_\sigma(\sigma)$ dependence proposed in [3], instead of τ_σ. As a result, Eq. (2) takes the form:

$$\tau_p \dot{\sigma} = -\sigma\left(1 + \frac{\theta^{-1} - 1}{1 + \sigma/\alpha}\right) + g_\Theta \varepsilon \tag{30}$$

where the relaxation time for the plastic mode $\tau_p = \eta_\sigma / \Theta$ is introduced ($\eta_\sigma \equiv \tau_\sigma G$ is the effective viscosity, Θ is the hardening factor), $\theta = \Theta / G < 1$ is the parameter describing the ratio of tilts for the deformation curve in the plastic and the Hookean domains, $g_\Theta = G^2 / \Theta G_0$ and $\alpha = \sigma_p / \sigma_s$ are the constants. Then, within the framework of approximation (6) the system (30), (3), and (4), as well as above, is reduced to equation (cf. (8)):

$$m\ddot{\sigma} + \gamma(\sigma)\dot{\sigma} = f(\sigma) + \phi(\sigma)\lambda(t) \tag{31}$$

where the coefficient of friction γ, force f, amplitude of noise ϕ, and parameter m are defined by expressions

$$\gamma(\sigma) \equiv \frac{1}{g_\Theta}\left[\tau_\varepsilon\left(1 + \frac{\theta^{-1} - 1}{(1 + \sigma/\alpha)^2}\right) + \tau_p\left(1 + \sigma^2\right)\right], \tag{32}$$

$$f(\sigma) \equiv \sigma\left[T_e - 1 - \frac{1}{g_\Theta}\left(\frac{\theta^{-1} + \sigma/\alpha}{1 + \sigma/\alpha}\right)\right] - \sigma^3\left[\frac{1}{g_\Theta}\left(\frac{\theta^{-1} + \sigma/\alpha}{1 + \sigma/\alpha}\right) - 1\right], \tag{33}$$

$$\phi(\sigma) \equiv \sigma, \; m \equiv \frac{\tau_p \tau_\varepsilon}{g_\Theta}. \tag{34}$$

According to the effective potential method [6], [12] – [14] we will obtain the Focker–Plank equation (10) with coefficients $D^{(1)}$ and $D^{(2)}$:

$$D^{(1)} = \frac{1}{\gamma}\left\{\sigma\left[T_e - 1 - \frac{1}{g_\Theta}\left(\frac{\theta^{-1} + \sigma/\alpha}{1 + \sigma/\alpha}\right)\right] + \sigma^3\left[1 - \frac{1}{g_\Theta}\left(\frac{\theta^{-1} + \sigma/\alpha}{1 + \sigma/\alpha}\right)\right] - \right.$$
$$\left. - I\sigma\tau_\lambda - \frac{2I\sigma}{\gamma^2}\left[\gamma - \frac{\sigma}{g_\Theta}\left(\frac{\tau_\varepsilon\left(1 - \theta^{-1}\right)}{(1 + \sigma/\alpha)^3 \alpha} + \sigma\tau_p\right)\right]\right\}, \tag{35}$$

$$D^{(2)} = \frac{I\sigma^2}{\gamma}\left[\gamma^{-1} + 2\tau_\lambda\right]. \tag{36}$$

In this case the more complex form for probability is observed and, as a result, the phase diagrams and portraits are more complex. Here, five different modes of friction are realized. Consider each of them separately. The $P(\sigma)$ dependences are shown in Fig. 3 for the different modes of friction.

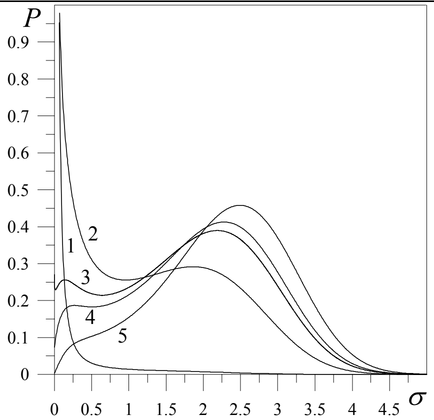

Fig. 3. The distribution function of shear stress for the first-order transition at $\tau_p = \tau_\varepsilon = \tau_\lambda = 0{,}1$, $\theta^{-1} = 7$, $\alpha = 0{,}3$, $g_\theta = 0.4$, $I = 4{,}5$. The curves 1 – 5 correspond to the temperatures $T_e = 16, 21, 23.25, 24, 26$, respectively

Curves 1 – 5 correspond to the regions of dry (DF), stick-slip (SS), stick-slip and sliding (SS+SF), metastable and stable sliding (MSF+SF), and sliding friction (SF). The phase portraits are presented in Fig. 4 meeting the curves of Fig. 3.

The phase portrait of dry friction region (DF) is similar to that is inherent in continuous transformation (Fig. 2, a). It implies, that DF regions are equivalent at the taking into account of the modulus defect and without it.

Phase portrait describing the region of stick-slip friction (SS) is similar to the characteristic one for the SS region at continuous transformation (Fig. 2, b). Basic their difference is that here the trajectories around focus are considerably more elongated along both coordinates axes. It means the greater stability of sliding friction.

The most complex region (SS+SF) is represented by the phase portrait shown in Fig. 4, a. The five singular points are realized here: *D*, saddles *N*, N', stable focuses *F*, F'. As well as above, saddles correspond to the minimums of $P(\sigma)$ dependence. Point *D* meets the solidlike state of lubricant. Stable focus *F* determines the first non-zero maximum of probability. It is apparent that the oscillations are weakly pronounced around this point. In this mode lubricant represents the very viscous liquid, because in such type of fluid at presence of noise the oscillations are damped strongly. Actually, the point *F* corresponds to the small values of stress, and with its decreasing the lubricant becomes more viscous, and at $\sigma = 0$ it is transformed into the solidlike state. Thus, using phase portraits it is possible to give explanation to that the liquidlike state of lubricant, but not the solidlike one, corresponds to the large values of shear stress.

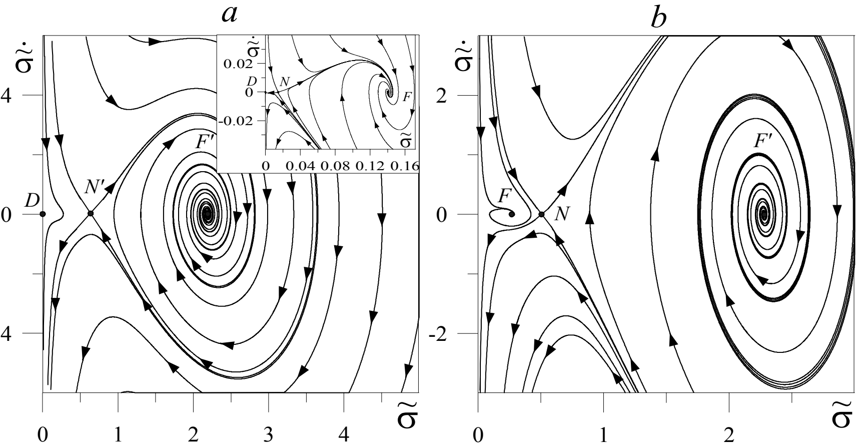

Fig. 4. The phase portraits corresponding to the parameters of Fig. 3: *a* – SS+SF mode corresponds to the curve 3 in Fig. 3; *b* – MSF+SF – curve 4 in Fig. 3

Focus F' meets the second non-zero maximum of $P(\sigma)$ function, and there are oscillations with large amplitude around it. This implies the fluidlike state of lubricant, and accordingly, sliding. This point is on large distance along abscissas axis from all others ones. This mode of friction is most probable only, since points *D* and *F*, corresponding to the dry and metastable sliding friction, have large stability and probability of realization also. From here the conclusion follows, that the system can undergo periodic transitions (stick-slip) between the modes corresponding to the points *D*, *F*, and F'. Since these modes are stable and separated by the pronounced minimums of distribution function $P(\sigma)$ (by saddles), the transitions between them is necessary to expect after large intervals of time.

The phase portrait of MSF+SF region is represented in Fig. 4b. There are three singular points – stable focuses *F*, F', and saddle *N*. The latter is similar to the described above saddles and meets the minimum of probability dependences on stress. Point *F* corresponds to the first maximum of distribution, which describes metastable sliding mode (MSF), and F' – to the second maximum, which defines stable sliding (SF). There are only insignificant oscillations around the focus *F*, however lubricant in this mode is less viscous liquid than in vicinity of point F' in Fig. 4a. At the origin of coordinates the singular point is absent, and the dry friction is not realized. Focus F' is similar to described in Fig. 4a, however, it's "attraction" domain is more stretched along both axes, that means the larger fluidity of lubricant and stability of this mode. Therefore in comparison with the previous case here the arising of sliding friction is more probable (SF).

SF region is represented by the phase portrait, which is similar to described at continuous transformation (Fig. 2c). Here, the one stable focus is realized *F* representing stable sliding friction (SF) characterized by oscillations in it's vicinity. The basic difference is that in this case oscillations take place with large amplitude, that implies strong fluidity of lubricant and pronounced stability of such mode. However, as well as in all above considered situations, in course of time in lubricant the stationary shear stress is set corresponding to the maximum of the initial distribution $P(\sigma)$.

In basic equations (2) – (4) the shear stress σ stands in the first power. However, in general case it's exponent a may be not integer, but fractional:

$$\tau_\sigma \dot{\sigma} = -\sigma^a + g\varepsilon , \tag{37}$$

$$\tau_\varepsilon \dot{\varepsilon} = -\varepsilon + (T - 1)\sigma^a , \tag{38}$$

$$\tau_T \dot{T} = (T_e - T) - \sigma^a \varepsilon + \sigma^{2a} + \lambda(t) . \tag{39}$$

Taking into account the additive noises of shear stress and strain, and the temperature of lubricant film it has been shown [5], that such system describes the self-similar mode for which the characteristic scale of shear stress is absent [16]. Such regime is determined by the homogeneous distribution function

$$P(y) = y^{-2a} P(\sigma), \quad y = \sigma\sigma_s . \tag{40}$$

In particular, the value $2a = 1.5$ corresponds to the self-organized criticality mode, at which, unlike the phase transition, the process of self-organization does not require the external influence ($T_e = 0$) and occurs spontaneously [5, 17].

The study of equations (37) – (39) shows that phase portraits for similar regions repeat above considered qualitatively. Thus, there is one substantial difference. The fractional Lorenz system at $a \neq 1$ and $I \neq 0$ results in presence of the point *D* in phase portraits, which corresponds to the solidlike state of lubricant. In addition, variation of a leads to the complication of $P(\sigma)$ dependence, and as a result, to more complex form of phase portraits. Within the limits of determined friction mode at decreasing of a the increase of abscissas of stable focuses is observed. Consequently, the weakening of fractional feedbacks in the Lorenz-type models result in the increase of lubricant fluidity and reducing of friction. However, in the systems described by fractional exponent a the dry friction is realized always. Thus, it is impossible simply to assert that such systems more preferable to friction decrease than linear systems.

The above consideration shows that increase of temperature of frictional surfaces T_e, at presence of colored noise of lubricant temperature, can be accompanied by self-organization of elastic and thermal fields leading to the mode of sliding friction. At setting of the sliding friction mode in the system the damping oscillations arise in the process of which the shear stress relaxes to the stationary value fixed by probability distribution. The amplitude of these oscillations increases with growth of stationary values of shear stress. It means that large shear stress σ corresponds to the liquidlike structure of lubricant. The solidlike state of lubricant is described by the singular point *D* at the origin of coordinates that has complex character of stability and corresponds to divergency of probability $P(\sigma)$. The oscillations near this point are absent.

For description of first-order transition the shear modulus defect is taken into account. It is shown that the change of value of the friction surfaces temperature T_e can transform the system from the mode of dry friction to the sliding one. At that the latter arises at two values of shear stress. Accordingly, the three singular points appear in phase portraits that define the stationary values of stress – non-standard point *D* at zero stress, and two stable focuses at non-zero ones. The interrupted (stick-slip) mode of friction can be realized as a result of transitions between solidlike, metastable and stable liquidlike lubricant states which are described by zero and non-zero singular points. Taking into consideration the nonlinear relaxation of shear stress and fractional feedbacks in the Lorentz system it has been shown that in phase portraits the singular point D is realized always, which corresponds to the solidlike state of lubricant and dry friction.

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ВПЛИВ КОРЕЛЯЦІЙ ТЕМПЕРАТУРИ НА ФАЗОВУ КІНЕТИКУ ГРАНИЧНОГО ТЕРТЯ

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В рамках моделі Лоренця для в'язкопружного середовища досліджено плавлення ультратонкої плівки мастила в процесі тертя між атомарно плоскими поверхнями. Флуктуації температури плівки мастила описують процесом Орнштейна–Уленбека. Побудовані фазові портрети, які відповідають різним областям динамічної фазової діаграми і визначають кінетику системи. Показано, що особлива точка, яка відповідає сухому тертю, має невизначений характер стійкості. Інші найімовірніші стани системи, що відповідають стійкому і метастабільному рідинному тертю, представлені на фазових портретах стійкими фокусами. Тому в системі можуть відбуватися затухаючі коливання до стійких станів. Велике продовження фазових траєкторій біля фокусів уздовж осей фазового портрета означає стійкість рідинного тертя. Оскільки максимуми функції розподілу, що відповідають стаціонарним режимам тертя, відокремлені один від одного вираженими мінімумами, переходи між ними відбуваються через великі інтервали часу.

Ключові слова: в'язкопружне середовище, фазовий портрет, переривчастий режим тертя.

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